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Relational Quantum Mechanics

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I suggest that the common unease with taking quantum mechanics as a fundamental description of nature (the "measurement problem") could derive from the use of an incorrect notion, as the unease with the Lorentz transformations before Einstein derived from the notion of observer-independent time. I suggest that this incorrect notion that generates the unease with quantum mechanics is the notion of "observer-independent state" of a system, or "observer-independent values of physical quantities." I reformulate the problem of the "interpretation of quantum mechanics" as the problem of deriving the formalism from a set of simple physical postulates. I consider a reformulation of quantum mechanics in terms of information theory. All systems are assumed to be equivalent, there is no observer-observed distinction, and the theory describes only the information that systems have about each other; nevertheless, the theory is complete.

1. A REFORMULATION OF THE PROBLEM OF THE "INTERPRETATION OF QUANTUM MECHANICS"

In this paper, I discuss a novel view of quantum mechanics. This point of view is not antagonistic to current ones, such as the Copenhagen (Heisenberg, 1927; Bohr, 1935), consistent-histories (Griffiths, 1984; Omnès, 1988), decohered-histories (Gell-Mann and Hartle, 1990), many-worlds (Everett, 1957; Wheeler, 1957; DeWitt, 1970), quantum-event (Hughes, 1989), or many-minds (Albert and Loewer, 1988, 1989; Lockwood, 1989; Donald, 1990) interpretations, but rather combines and complements aspects of them. This paper is based on a critique of a notion generally assumed uncritically. As such, it bears a vague resemblance with Einstein's discussion of special relativity, which is based on the critique of the notion of absolute simultaneity. The notion rejected here is the notion of absolute, or observer-independent, state of a system; equivalently, the notion of observer-independent values of

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physical quantities. The thesis of the present work is that by abandoning such a notion (in favor of the weaker notion of state—and values of physical quantities—*relative* to something), quantum mechanics makes much more sense. This conclusion derives from the observation that the experimental evidence at the basis of quantum mechanics forces us to accept that distinct observers give different descriptions of the same events. From this, I shall argue that the notion of observer-independent state of a system is inadequate to describe the physical world beyond the $\hbar \rightarrow 0$ limit, in the same sense in which the notion of observer-independent time is inadequate to describe the physical world beyond the $c \rightarrow \infty$ limit. I then consider the possibility of replacing the notion of absolute state with a notion that refers to the relation between physical systems.

The motivation for the present work is the commonplace observation that in spite of the seven decades since the discovery of quantum mechanics, and in spite of the variety of approaches developed with the aim of clarifying its content and improving the original formulation, quantum mechanics maintains a remarkable level of obscurity. It has even been accused of being unreasonable and unacceptable, even inconsistent, by world-class physicists (for example, Newman, 1993). My point of view in this regard is that quantum mechanics synthesizes most of what we have learned so far about the physical world: The issue is thus not to replace or fix it, but rather to understand *what* it actually says about the world; or, equivalently, what precisely we have learned from experimental microphysics.

Still, it *is* difficult to overcome the sense of unease that quantum mechanics communicates. The troubling aspect of the theory assumes different faces within different interpretations, and therefore a complete description of the problem can only be based on a survey of already proposed solutions. Here, I do not attempt such a survey; for a classical review see d'Espagnat (1971); a more modern survey is in the first chapters of Albert (1992), or, in compact, Butterfield (1995). The unease is expressed, for instance, in the objections the supporters of each interpretation raise against other interpretations. For example: Is there something "physical" happening when the wave function "collapses"? Is it really possible that observer and measurement, including wave function reduction, cannot be described in Schrödinger evolution terms? How can a classical world emerge from a quantum reality? If somebody would prepare me in a quantum superposition of two macroscopic states, how would I feel? If the Planck constant were 25 orders of magnitude larger, what would the world look like? "Who" or "what" determines the *family* of consistent histories that describe a set of events, and can this "chooser" be described quantum mechanically? And so on. Some of these questions are perhaps naive or ill-posed, but the fact that they are regularly asked, and that no interpretation of the theory has so far succeeded in answering all objections satisfactorily, indicates, I believe, that the problem of the interpretation of quantum mechanics has not been fully disentangled. This unease, and the variety of interpretations of quantum mechanics that it has generated, are sometimes denoted the "measurement problem." In this paper, I address this sense of unease, and propose a way out.

The paper is based on two ideas:

1. That this unease may derive from the use of a concept which is inappropriate to describe the physical world at the quantum level. I shall argue that this concept is the concept of observer-independent state of a system, or, equivalently, the concept of observer-independent values of physical quantities.
2. That quantum mechanics will cease to look puzzling only when we will be able to *derive* the formalism of the theory from a set of simple physical assertions ("postulates," "principles") about the world. Therefore, we should not try to *append* a reasonable interpretation to the quantum mechanics *formalism*, but rather to *derive* the formalism from a set of experimentally motivated postulates.

The reasons for exploring such a strategy are illuminated by an obvious historical precedent: special relativity. I shall make use of this analogy for explanatory reasons, in spite of the evident limits of the simile.

Special relativity is a well-understood physical theory, appropriately ascribed to Einstein's celebrated paper of 1905. It is interesting in this context, however, to recall the well-known fact that the formal content of special relativity is entirely coded in the Lorentz transformations, which were written by Lorentz, not by Einstein, and several years before 1905. So, what was Einstein's contribution? It was to understand the physical meaning of the Lorentz transformations (and more, but this is what is of interest here). We could say, admittedly in a provocative manner, that Einstein's contribution to special relativity was the *interpretation* of the theory, not its *formalism*: the formalism already existed.

Lorentz transformations were widely discussed at the beginning of the century, and their *interpretation* was much debated. In spite of the recognized fact that they represent an extension of the Galilean group compatible with Maxwell theory, the Lorentz transformations were perceived as rather unreasonable and unacceptable as a fundamental spacetime symmetry transformation, even inconsistent, before 1905; a situation that may recall the present state of quantum mechanics. The physical interpretation proposed by Lorentz himself (and defended by Lorentz long after 1905) was a physical contraction of moving bodies, caused by complex and unknown electromagnetic interaction between the atoms of the bodies and the ether. It was a quite unattractive interpretation (and remarkably similar to certain interpretations of wave function collapse as presently investigated!). Einstein's 1905 paper suddenly clarified the matter by pointing out the reason for the unease of taking the Lorentz transformations "seriously": the implicit use of a concept (observer-independent time) inappropriate to describe reality when velocities are high. Equivalently: a common deep assumption about reality (simultaneity is observer-independent) which is physically incorrect. The unease with the Lorentz transformations derived from a conceptual scheme in which an *incorrect notion*—absolute simultaneity—was assumed, yielding all sorts of paradoxical situations. Once this notion was removed, the physical interpretation of the Lorentz transformations stood clear and special relativity is now universally considered rather uncontroversial.

Here I consider the hypothesis that all "paradoxical" situations associated with quantum mechanics—such as the famous and unfortunate half-dead Schrödinger cat (Schrödinger, 1935)—may derive from some analogous *incorrect notion* that we use in thinking about quantum mechanics. (Not in using quantum mechanics, since we seem to have learned to use it in a remarkably effective way.) The aim of this paper, therefore, is to hunt for this *incorrect notion*, with the hope that by exposing it clearly to public contempt, we could free ourselves from the present unease with our best present theory of motion, and fully understand what the theory is asserting about the world.

Furthermore, Einstein was so persuasive with his interpretation of the Lorentz equations because he did not *append* an interpretation to them: rather, he *rederived* them, starting from two "postulates" with terse physical meaning—equivalence of inertial observers and universality of the speed of light—taken as facts of experience. This rederivation unraveled the physical content of the Lorentz transformations and provided them with a solid *interpretation*. I would like to suggest that in order to grasp the full physical meaning of quantum mechanics, a similar result should be achieved: Find a small number of simple statements about nature—which may perhaps be apparently contradictory, as the two postulates of special relativity are—with clear physical meaning, from which the formalism of quantum mechanics could be *derived*. To my knowledge, such a derivation has not yet been achieved. In this paper, I do not achieve such a result in a satisfactory manner, but I discuss a possible reconstruction scheme.

The program outlined is thus to do for the formalism of quantum mechanics what Einstein did for the Lorentz transformations: (i) Find a set of simple assertions about the world, with clear physical meaning, that we know are experimentally true (postulates); (ii) analyze these postulates, and show that from their conjunction it follows that certain common assumptions about the world are incorrect; (iii) derive the full formalism of quantum mechanics from these postulates. I expect that if this program could be completed, we would at long last begin to agree that we have "understood" quantum mechanics.

In Section 2 I analyze the measurement process as described by two distinct observers. This analysis leads to the main idea: the observer dependence of state and physical quantities, and to recognize a few key concepts in terms of which, I would like to suggest, the quantum mechanical description of reality "makes sense." Prominent among these is the concept of information (Shannon, 1949; Wheeler, 1988, 1989, 1992). In Section 3 I switch from an inductive to a (very mildly) deductive mode, and I put forward a set of notions, and a set of simple physical statements, from which the formalism of quantum mechanics can be reconstructed. I denote these statements as "postulates," at the risk of misunderstanding: I do not claim any mathematical or philosophical rigor, nor completeness, in the derivation—supplementary assumptions are made along the way. I am not interested here in a formalization of the subject, but only in grasping its "physics." In particular, ideas and techniques for the reconstruction are borrowed from quantum logic research, but motivations and spirit are different. Finally, in Section 4 I discuss the picture of the physical world that has emerged, and attempt an evaluation. In particular, I compare the approach I have developed with some currently popular interpretations of quantum mechanics, and argue that the differences between those disappear if the results presented here are taken into account.

In order to prevent the reader from channeling his or her thoughts in the wrong direction, let me anticipate a remark. By using the word "observer" I do not make any reference to conscious, animate, or computing, or in any other manner special, systems. I use the word "observer" in the sense in which it is conventionally used in Galilean relativity when we say that an object has a velocity "with respect to a certain observer." The observer can be any physical object having a definite state of motion. For instance, I say that my hand moves at a velocity v with respect to the lamp on my table. Velocity is a relational notion (in Galilean as well as in special relativistic physics), and thus it is always (explicitly or implicitly) referred to "something"; it is traditional to denote this something as the "observer," but it is important in the following discussion to keep in mind that the "observer" can be a table lamp. Similarly, I use information theory in its original (Shannon) form, in which information is a measure of the number of states in which a system can be—or in which several systems whose states are physically constrained (correlated) can be. Thus, a pen on my table has information because it points in this or that direction. We do not need a human being, a cat, or a computer to make use of this notion of information.

2. QUANTUM MECHANICS IS A THEORY ABOUT INFORMATION

In this section, a preliminary analysis of the process of measurement is presented, and the main ideas and arguments are introduced. Throughout this section, standard quantum mechanics and standard interpretation—by which I mean for instance: formalism and interpretation in Dirac (1930) or Messiah (1958)—are assumed.

2.1. The Third-Person Problem

Consider an observer O (Observer) which makes a measurement on a system S (System). For the moment we may think of O as a classical macroscopic measuring apparatus, including or not including a human being—this being irrelevant for what follows. Assume that the quantity being measured, say q, takes two values, 1 and 2; and let the states of the system S be described by vectors (rays) in a two (complex)-dimensional Hilbert space H_S. Let the two eigenstates of the operator corresponding to the measurement of q be $|1\rangle$ and $|2\rangle$. As is well known, if S is in a generic normalized state $|\psi\rangle = \alpha|1\rangle + \beta|2\rangle$, where α and β are complex numbers and $|\alpha|^2 + |\beta|^2 = 1$, then O can measure either one of the two values 1 and 2, the respectively probabilities being $|\alpha|^2$ and $|\beta|^2$.

Let us assume that in a *given specific measurement* the outcome of the measurement is 1. From now on, we concentrate on describing *this* specific experiment, which we denote as **E**. The system S is affected by the measurement, and at a time $t = t_2$ after the measurement, the state of the system is $|1\rangle$. In the physical sequence of events **E**, the states of the system at t_1 and t_2 are thus

$$\begin{gathered} t_1 \to t_2 \\ \alpha|1\rangle + \beta|2\rangle \to |1\rangle \end{gathered} \tag{1}$$

This is the standard account of a measurement according to quantum mechanics.

Let us now consider this same sequence of events **E**, as described by a *second* observer, which we refer to as P: P is an observer different from O. I shall refer to O as "he" and to P as "she." P may describe a system formed by S and O. Therefore she views both S and O as subsystems of the larger S–O system she is considering. Again, we assume P uses conventional quantum mechanics. We also assume that P does not perform any measurement on the S–O system during the t_1–t_2 interval, but that she knows the initial states of both S and O, and is thus able to give a quantum mechanical description of the set of events **E**. She describes the system S by means of the Hilbert space H_S considered above, and O by means of a Hilbert space H_O. The S–O system is then described by the tensor product $H_{SO} = H_S \otimes H_O$. As has become conventional, let us denote the vector in H_O that describes the state of the observer O at $t = t_1$ (prior to the measurement) as $|\text{init}\rangle$. The physical process during which O measures the quantity q of the system S

implies a physical interaction between O and S. In the process of this interaction, the state of O changes. If the initial state of S is $|1\rangle$ (resp. $|2\rangle$) (and the initial state of O is $|\text{init}\rangle$), then $|\text{init}\rangle$ evolves to a state that we denote as $|O1\rangle$ (resp. $|O2\rangle$). We think of $|O1\rangle$ (resp. $|O2\rangle$) as a state in which "the position of the hand of a measuring apparatus points toward the mark '1' (resp. '2')." It is not difficult to construct model Hamiltonians that produce evolutions of this kind, and that can be taken as models for the physical interactions that produce a measurement. Let us consider the actual case of the experiment **E**, in which the initial state of S is $|\psi\rangle = \alpha|1\rangle + \beta|2\rangle$. The initial full state of the S–O system is then $|\psi\rangle \otimes |\text{init}\rangle = (\alpha|1\rangle + \beta|2\rangle) \otimes |\text{init}\rangle$. As well known, the linearity of quantum mechanics implies

$$\begin{gathered} t_1 \rightarrow t_2 \\ (\alpha|1\rangle + \beta|2\rangle) \otimes |\text{init}\rangle \rightarrow \alpha|1\rangle \otimes |O1\rangle + \beta|2\rangle \otimes |O2\rangle \end{gathered} \tag{2}$$

Thus, at $t = t_2$ the system S–O is in the state $(\alpha|1\rangle \otimes |O1\rangle + \beta|2\rangle \otimes |O2\rangle)$. This is the conventional description of a measurement as a physical process (von Neumann, 1932).

I have described an actual physical process **E** taking place in a real laboratory. Standard quantum mechanics requires us to distinguish system from observer, but it allows us freedom in drawing the line that distinguishes the two. The peculiarity of the above analysis is just the fact that this freedom has been exploited in order to describe the same sequence of physical events in terms of two different descriptions. In the first description, equation (1), the line that distinguishes system from observer is set between S and O. In the second, equation (2), it is between S–O and P. I recall that we have assumed that P is not making a measurement on the S–O system; there is no physical interaction between S–O and P during the t_1–t_2 interval. P may make measurements at a later time t_3: if she measures the value of q on S and the position of the hand on O, she finds that the two agree, because the first measurement collapses the state into one of the two factors of (2), leaving the second measurement fully determined to be the consistent value. Thus, we have two descriptions of the physical sequence of events that we have denoted **E**: The description (1) given by the observer O and the description (2) given by the observer P. The point I would like to emphasize here is that these are two distinct *correct* descriptions of the same sequence of events **E**. In the O description, the system S is in the state $|1\rangle$ and the quantity q has value 1. According to the P description, S is not in the state $|1\rangle$ and the hand of the measuring apparatus does not indicate "1."

Thus, I come to the observation on which the rest of the paper relies.

Main Observation. In quantum mechanics different observers may give different accounts of the same sequence of events.

For a very similar conclusion, see Zurek (1982). In the rest of the work I explore the consequences of taking this observation fully into account. Since this observation is crucial, I now pause to discuss and reject various potential objections to the main observation. The reader who finds the above observation plausible may skip this rather long list of objections and jump to Section 2.3.

2.2. Objections to the Main Observation, and Replies

Objection 1. Whether the account (1) or the account (2) is correct depends on which kind of system O happens to be. There are systems that induce the collapse of the wave function. For instance, if O is macroscopic, (1) is correct, if O is microscopic (2) is correct.

This implies that O cannot be described as a genuine quantum system. Namely there are "special systems" that do not obey conventional quantum mechanics, but are "intrinsically classical" in that they produce collapse of the wave functions—or actualization of the values of quantities. This idea underlies a variety of old and recent attempts to unravel the quantum puzzle, the special systems being, for instance, gravity (Penrose, 1989), or minds (Albert and Loewer, 1988), or macroscopic systems (Bohr, 1949). If we accept this idea, we have to separate reality into two kinds of systems: quantum mechanical systems on the one hand, and special systems on the other. Bohr claims explicitly that we have to renounce giving a full quantum mechanical description of the classical world (Bohr, 1949). This is echoed in such texts as Landau and Lifshitz (1977). Wigner pushes this view to the extreme consequences and distinguishes material systems (observed) from consciousness (observer) (Wigner, 1961). Here, on the contrary, I wish to assume the following:

Hypothesis 1. All systems are equivalent: Nothing *a priori* distinguishes macroscopic systems from quantum systems. If the observer O can give a quantum description of the system S, then it is also legitimate for an observer P to give a quantum description of the system formed by the observer O.

Of course, I have no proof of Hypothesis 1. However, let me illustrate my motivations for holding it. I am suspicious toward attempts to introduce special nonquantum and not-yet-understood systems or special new physics to alleviate the strangeness of quantum mechanics: they look very much like Lorentz's attempt to postulate a mysterious interaction that Lorentz-contracts physical bodies "for real"—something that we now perceive as ridiculous, in the light of Einstein's clarity. Virtually all those views modify quantum mechanical predictions, in spite of common statements to the contrary: if at t_2 the state is as in (1), then P can never detect interference terms between the two branches in (2), contrary to quantum theory predictions. Admittedly, these discrepancies are likely to be minute, as shown by the beautiful discovery of the physical mechanism of decoherence (Zurek, 1981; Joos and Zeh, 1985), which *saves the phenomena*. But they are nevertheless different from zero, and thus observable (more on this later). I am inclined to trust that a sophisticated experiment able to detect those minute discrepancies will fully vindicate quantum mechanics against any distortion due to postulated intrinsic classicality of specific systems. The question is experimentally decidable; so we shall see. Second, I do not like the idea that the present extremely successful theory of motion can only be understood in terms of failures that are yet to be detected. Finally and most importantly, I maintain it is reasonable to remain committed, up to compelling disproof, to the golden rule that all physical systems are equivalent in respect to mechanics: this golden rule has proven so overwhelmingly successful that I am not ready to dismiss it as long as there is another way out.

Objection 2. What the discussion indicates is that the quantum state is different in the two accounts, but the quantum state is a fictitious nonphysical mental construction; the physical content of the theory is given by the outcomes of the measurements.

Indeed, one can take the view that outcomes of measurements are the only physical content of the theory, and the quantum state is a secondary theoretical construction. This is the way I read Heisenberg (1927) and van Fraassen (1991). According to this view, anything in between two measurement outcomes is like the "nonexisting" trajectory of the electron, to use Heisenberg's vivid expression, of which there is nothing to say. I am very sympathetic with this view, which plays an important role in Section 3. This view, however, does not circumvent the main observation for the following reason. The account (2) states that there is nothing to be said about the value of the quantity q of S at time t_2. Observer P claims that at $t = t_2$ the quantity q does not have a determined value. On the other hand, O claims that, at $t = t_2$, q has the value 1. From which the main observation follows again.

Objection 3. As before (only outcomes of measurements are physical), but the truth of the matter is that P is right and O is wrong.

This is undefendable. Since all physical experiments of which we know can be seen as instances of the S–O measurement, this would imply that not a single outcome of measurement has ever been obtained. If so, how could we have learned quantum theory?

Objection 4. As before (only outcomes of measurements are physical), but the truth of the matter is simply that O is right and P is wrong.

If P is wrong, quantum mechanics cannot be applied to the S–O system (because her account is a straightforward implementation of textbook quantum mechanics). Thus this objection predicts discrepancies, so far never observed, with quantum mechanical predictions, which include observable interference effects between the two terms of (2).

Objection 5. As before, but under the assumption that O is macroscopic. Then the interference terms mentioned become extremely small because of decoherence effects. If they are small enough, they are unobservable, and thus $q = 1$ becomes an absolute property of S, which is true and absolutely determined, albeit unknown to P, who could measure it anytime, and would not see interference effects.

Again, strictly speaking this is wrong, because decoherence depends on which subsequent observation P does. Therefore, the property $q = 1$ of S would become an absolute property at time t_2 or not, according to which subsequent properties of S the observer (P) considers. This is the reason the idea of exploiting physical decoherence for interpreting quantum mechanics has evolved into the consistent and decoherent histories interpretations, where probabilities are (consistently) assigned to histories, and not to single outcomes of measurements within a history. [See, however, the discussion on the no-histories slogan in Butterfield (1995).]

Objection 6. There is no collapse. The description (1) is not correct, because "the wave function never really collapses." The account (2) is the correct one. There are no values assigned to classical properties of system, but only quantum states.

If so, then the observer P cannot measure the value of the property q either, since (because of assumption) there are no values assigned to classical properties but only quantum states; thus the quantity q *never* has a value. But we do describe the world in terms of "properties" that the systems have and values assumed by various quantities, not in terms of states in the Hilbert space vector. In a description of the world purely in terms of quantum states, the systems never have definite properties and I do not see how to match the description with any observation. For a detailed elaboration of this argument, which I view as very strong, see Albert (1992).

Objection 7. There is no collapse. The description (1) is not correct, because "the wave function never really collapses." The account (2) is the correct one. The values assigned to classical properties are different from branch to branch.

This is a form of Everett's view (Everett, 1957), which entails the idea that when we measure the electron's spin being up, the electron spin is also and simultaneously down "in some other branch"—or "world," hence the "many-world" denomination of this view. The property of the electron of having spin up is then not absolutely true, but just true relative to "this" branch, namely we simply have a new "parameter" for expressing contingency: "which branch" is a new "dimension" of indexicality, in addition to the familiar ones as "which time" and "which place." Thus, the state of affairs of the example is that, at t_2, q *has* value 1 in one branch and *has* value 2 in the other; the two branches being theoretically described by the two terms in (2). This is a fascinating idea that has recently been implemented in a variety of diverse incarnations. Traditionally, the idea has been discussed in the context of the notion of apparatus, namely a distinguished set of subsystems of the universe, and a distinguished quantity on such an apparatus—the preferred basis. Such a (collection of) preferred apparatuses and preferred bases are needed in order to define branching, and thus in order to have assignment of values (Butterfield, 1995); the view has recently branched (!) into the many-mind interpretations, where the distinguished subsystems are related to various aspects of the human brain [see Butterfield (1995) for a recent discussion]. All these versions of Everett's idea violate Hypothesis 1, and thus I am not concerned with them here. Alternatively, there are versions of Everett's idea that reject the specification of a preferred apparatus and preferred basis, and in which the branching itself is indexed by an arbitrarily chosen system playing the role of apparatus and an arbitrarily chosen basis. To my knowledge, the only elaborated versions of this view which avoid the difficulties mentioned in Objection 5 have evolved into the histories formalisms considered below.

Objection 8. What is absolute and observer independent is the probability of a sequence $A_1, \ldots, A_n$ of property ascriptions (such that the interference terms mentioned above are extremely small—decoherence); this probability is independent of the existence of any observer measuring these properties.

This is certainly correct, and, in fact, this observation is at the root of the histories interpretations of quantum mechanics (Griffiths, 1984; Omnès, 1988; Gell-Mann and Hartle, 1990). However, this view confirms the observation above that different observers give different accounts of the same sequence of events, for the following—often overlooked—reason. The beauty of the histories interpretations is the fact that the probability of *a sequence of outcomes* within a consistent family of sequences does not depend on the observer, precisely as it does not in classical mechanics. One can be content with this powerful aspect of the theory and consistently stop here. However, the description of a single property description depends on the choice of later properties considered as well as on the choice of the consistent *family* of histories used to describe a sequence of events. The observer makes a choice in picking up a family of alternative histories in terms of which he or she describes the system. Unlike classical mechanics, this choice is such that the same property can be true or not, according to the family chosen (Griffiths, 1993; Hartle, 1994). Now, the observer (O), too, is a physical system (even Murray Gell-Mann is a physical system). Unless we assume that the observer is an unphysical entity, we then are free to consider how a second observer would describe the events, and we are back to the problem above: the descriptions of the *same* sequence of events given by the two observers can be different. Classes of observers may agree on sets of outcomes (or interpret the differences as statistical ignorance), but there may always be other observers, perhaps observing the sequence of events *and* O, who have chosen a *different family* of consistent histories to describe the same sequence of events. Note that not just the probabilities, but even the actual description of what has happened in a concrete instance (see Note 5 below) can be different. Thus, the histories interpretations do not emphasize, but confirm the conclusion that if an observer O gives a description of a sequence of events, another observer—choosing a different family of histories—may give a different description of the same sequence. I shall return to this point in the last section.

In conclusion, it seems to me that whatever view of quantum theory (consistent with Hypothesis 1) one may hold, the main observation is inescapable. I thus proceed to the main point of this work.

2.3. Main Discussion

If different observers give different accounts of the same sequence of events, then each quantum mechanical description has to be understood as relative to a particular observer. Thus, a quantum mechanical description of a certain system (state and/or values of physical quantities) cannot be taken as an "absolute" (observer-independent) description of reality, but rather as a formalization, or codification, of properties of a system *relative* to a given observer. Quantum mechanics can therefore be viewed as a theory about the states of systems and values of physical quantities relative to other systems.

A quantum description of the state of a system S exists only if some system O (considered as an *observer*) is actually "describing" S, or, more precisely, has interacted with S. The quantum state of a system is always a state of that system with respect to a certain other system. More precisely: when we say that a physical quantity takes the value v, we should always (explicitly or implicitly) qualify this statement as: the physical quantity takes the value v with respect to the so and so observer. Thus, in the example considered in Section 2.1, q has value 1 *with respect to O*, but not to P. I am convinced that there is no way to escape this conclusion.

Therefore, I maintain that in quantum mechanics, "state" as well as "value of a variable"—or "outcome of a measurement"—are relational notions in the same sense in which velocity is relational in classical mechanics. We say "the object S has velocity v" meaning "with respect to a reference object O." Similarly, I maintain that "the system is in such a quantum state" or "$q = 1$" are always to be understood "with respect to the reference O." In quantum mechanics *all* physical variables are relational, as velocity is.

If quantum mechanics describes only relative information, one could consider the possibility that there is a deeper underlying theory that describes what happens "in reality." This is the thesis of the incompleteness of quantum mechanics [first suggested by Born (1926)!]. Examples of hypothetical underlying theories are hidden variables theories (Bohm, 1951; Belifante, 1973). Alternatively, the "wave-function-collapse-producing" systems can be "special" because of some non-yet-understood physics, which becomes relevant due to the large number of degrees of freedom (Ghirardi *et al.*, 1986; Bell, 1987), complexity (Hughes, 1989), quantum gravity (Penrose, 1989), or other factor.

As is well known, there are no indications on *physical* grounds that quantum mechanics is incomplete. Indeed, the *practice* of quantum mechanics supports the view that quantum mechanics represents the best we can say about the world at the present state of experimentation, and suggests that the structure of the world grasped by quantum mechanics is deeper, and not shallower, than the scheme of description of the world of classical mechanics. On the other hand, one could consider motivations on *metaphysical* grounds in support of the incompleteness of quantum mechanics. One could argue: "Since reality has to be real and universal, and the same for everybody, then a theory in which the description of reality is observer-dependent in certainly an incomplete theory." If such a theory were complete, our concept of reality would be disturbed.

But the way I have reformulated the problem of the interpretation of quantum mechanics in Section 1 should make us suspicious and attentive precisely to such kinds of arguments. Indeed, what we are looking for is *precisely* some "wrong general assumption" that we suspect to have, and that could be at the origin of the difficulty in understanding quantum mechanics. Thus, I discard here the thesis of the incompleteness of quantum mechanics and tentatively assume the following.

Hypothesis 2 (Completeness). Quantum mechanics provides a complete and self-consistent scheme of description of the physical world, appropriate to our present level of experimental observations.

The conjunction of Hypothesis 2 with the "Main Observation" of Section 2.1 and the discussion above leads to the following idea:

Quantum mechanics is a theory about the physical description of physical systems relative to other systems, and this is a complete description of the world.

The thesis of this paper is that this conclusion is not self-contradictory. If this conclusion is valid, then the "incorrect notion" at the source of our unease with quantum theory has been uncovered: it is the notion of true, universal, observer-independent description of the state of the world. If the notion of observer-independent description of the world is unphysical, a complete description of the world is exhausted by the relevant information that systems have about each other. Namely, there is neither an absolute state of the system, nor absolute properties that the system has at a certain time. Physics is fully relational, not just as far as the notions of "rest" and "motion" are considered, but with respect to any physical quantities. Accounts (1) and (2) of the sequence of events **E** are both correct, even if distinct: any time we talk about a "state" or "property" of a system, we have to refer these notions to a specific "observing" or "reference" system. Thus, I propose the idea that quantum mechanics indicates that the notion of a "universal" description of the state of the world, shared by all observers, is a concept which is physically untenable, on experimental grounds. It is valid only in the $\hbar \to 0$ approximation.[2]

Thus, the hypothesis on which I base this paper is that accounts (1) and (2) are both fully correct. They refer to different observers: (1) refers to O, while (2) refers to P. I propose to reinterpret every contingent statement about nature, as for instance, "the electron has spin up," "the atom is in the so and so excited state," the "spring is compressed," "the chair is here and not there," as elliptic expressions for relational assertions: "the electron has spin up *with respect to the Stern–Gerlach apparatus*," . . . , "the chair is here and not there *with respect to my eyes*," and so on. Quantum states, as well as values of physical quantities, make sense only when referred to a physical system (which I denote as the observer system, or reference system). A general physical theory is a theory about the state that physical systems have relative to each other. I explore and elaborate this possibility in this paper.

2.4. Relation Between Descriptions

An immediate issue raised by the multiplication of points of view induced by the relational notion of state and the values of physical quantities is the

[2] To counter objections based on intuition alone, it is perhaps worthwhile recalling the great resistance that the idea of fully relational notions of "rest" and "motion" encountered at the beginning of the scientific revolution; and the fact that we should perhaps be ready to accept that quantum mechanics (and general relativity) could well be in the course of triggering of a—not yet developed—revision of world views as far-reaching as the seventeenth century's [on this, see Rovelli (1995)].

problem of the relation between distinct descriptions of the same events. What is the relation between the value of a variable q relative to an observer O and the value of the same variable relative to a different observer? We do not like a solipsistic world of uncommunicative observers, nor, in any case, is this what quantum mechanics describes. Let me reconsider the example of Section 2.1. It is clear that there is some relation between the description of the world illustrated in (1) and in (2). More precisely, we may ask two questions: (i) Does P "know" that S "knows" the value of q? (ii) Does P know what is the value of q relative to O? I consider these two questions separately.

(i) Does P "know" that O has made a measurement on S at time t_2? The answer is a definite yes, for a number of reasons. First, the observer P has a full account of the events **E**, so it should be possible to interpret her description (2) as expressing the fact that O has measured S. Indeed this is possible: The key observations is that in the final state at t_2 in (2), the variables q (with eigenstates $|1\rangle$ and $|2\rangle$) and the pointer variable (with eigenstates $|O1\rangle$ and $|O2\rangle$) are correlated. From this fact, the observer P understands that the pointer variable in O has information about q. In fact, the state of S–O is the quantum superposition of two states: in the first, $(|1\rangle \otimes |O1\rangle)$, S is in the $|1\rangle$ state and the hand of the observer is correctly on the "1" mark. In the second, $(|2\rangle \otimes |O2\rangle)$, S is in the $|2\rangle$ state and the hand of the observer is, correctly again, on the "2" mark. In both cases, the hand of O is on the mark that correctly represents the state of the system. More precisely, P could consider an observable M that checks whether the hand of O indicates the correct state of S. If she measured M, then the outcome of this measurement would be "yes" with certainty, when the state of the S–O system is as in (2). The operator M is given by

$$M|1\rangle \otimes |O1\rangle = |1\rangle \otimes |O1\rangle, \qquad M|1\rangle \otimes |O2\rangle = 0 \qquad (3)$$
$$M|2\rangle \otimes |O2\rangle = |2\rangle \otimes |O2\rangle, \qquad M|2\rangle \otimes |O1\rangle = 0$$

where the eigenvalue 1 means "yes, the hand of O indicates the correct state of S," and the eigenvalue 0 means "no, the hand of O does not indicate the correct state of S." Thus, it is meaningful to say that, according to the P description of the events **E**, O "knows" the quantity q of S, or that he "has measured" the quantity q of S, and the pointer variable embodies the information.

The other question P may ask is: (ii) What is the outcome of the measurement performed by O? It is important not to confuse the statement "P knows that O knows the value of q" with the statement "P knows what O knows about q." Of course this is a consistent distinction common in everyday life (I know *that* you know the amount of your salary, but I do not know *what* you know about the amount of your salary). Now in general the observer P does not know "what is the value of the observable q that O has measured" [unless α or β in (2) vanishes]. The relation between the descriptions that different observers give of the same event is characterized by the fact that an observer with sufficient initial information may predict what the other observer has measured, but not the outcome of the measurement. Communication of measurements results is, however, possible (and fairly common!). P can measure the outcome of the measurement performed by O. She can, indeed, measure whether O is in $|O1\rangle$ or in $|O2\rangle$. Notice that there is a consistency condition to be fulfilled, which is the following: if P knows that O has measured q, and then she measures q, and then she measures what O has obtained in measuring q, consistency requires that the results obtained by P about the variable q and the pointer are correlated. Indeed, they are, as was first noticed by von Neumann, and is clear from (2). Thus, there is a satisfied consistency requirement in the notion of relative description discussed. This can be expressed in terms of standard quantum mechanical language: From the point of view of the P description:

The fact that the pointer variable in O has information about S (has measured q) is expressed by the existence of a correlation between the q variable of S and the pointer variable of O. The existence of this correlation is a measurable property of the O–S state.

Notice that representing the fact that (for P) "the pointer variable of O has information about the q variable in S" by means of the operator M resolves the well-known and formidable problem of defining the "precise moment" in which the measurement is performed, or the precise "amount of correlation" needed for a measurement to be established—see for instance Bacciagaluppi and Hemmo (1995). Such questions are not classical questions, but quantum mechanical questions, because whether or not O has measured S is not an absolute property of the O–S state, but a quantum property of the quantum O–S system, that can be investigated by P, and whose yes/no answers are, in general, determined only probabilistically. In other words: *imperfect correlation does not imply no measurement performed, but only a smaller than 1 probability that the measurement has been completed.*

2.5. Information

It is time to introduce the main concept in terms of which I propose to interpret quantum mechanics: information. In Section 2.3 I emphasized the relational character of any quantum mechanical assertion: the complete meaning of "$q = 1$" is "$q = 1$ relative to O." The main hypothesis here is that this relational character of physical statements is not due to incompleteness of quantum theory, but to the physical inapplicability of the notion of "observer-independent value of q" to the natural world. Now, one may ask, what is the nature of the relation between the variable q and the system O expressed in the statement "$q = 1$ relative to O"? In other words, does this relation have a comprehensible physical meaning? Can we analyze it in physical terms?

Let me begin by a purely lexical move. I will denote the relation between a physical quantity q of a system S and the observer system O with respect to which q has a certain value as "information." I will say "O has the information that $q = 1$" to mean "$q = 1$ relative to S." The use of this expression "information" emerges naturally in considering the example of Section 2.1, where the observing systems O and P are complex observers actively gathering information about S; but I ask the reader to temporarily suspend any evocative implication of the expression "information" at this stage. For the moment, "information" is just a word to denote the relational character of every contingent assertion about values of physical quantities or states of systems. The question addressed here is thus what is the physical meaning of the fact that O has information about the variable q?

If the statement "q has a value relative to O" or "O has information about q" has any comprehensible physical meaning at all, this meaning should be related to the contingent state of the S–O system. According to the main hypothesis here, asking about the observer-independent contingent state of the S–O system has no meaning. Therefore, it seems at first that we cannot understand what is the physical nature of the S–O relation. But this conclusion is too precipitous. Indeed, we can make statements about the state of the S–O system, provided that we interpret these statements as relative to a third physical system P. Therefore, it should be possible to understand what is the physical meaning of "q has a value relative to O" by considering the description that P gives (or could give) of the S–O system. This description is not in terms of classical physics, but in quantum mechanical terms; it is the one given in detail in Section 2.4. The result is that "S has information about q" means that there is a correlation between the variable q and the pointer variable in O.

This result provides a motivation for the use of the expression "information" because information is correlation. The notion of information I employ here should not be confused with other notions of information used in other contexts. I shall use here a notion of information that does not require distinction between human and nonhuman observers, systems that understand meaning or do not, very complicated or simple systems, and so on. As is well known, the problem of defining such a notion was brilliantly solved by Shannon (1949): in the technical sense of information theory, the amount of information is the number of the elements of a set of alternatives out of which a configuration is chosen. Information expresses the fact that a system is in a certain configuration, which is correlated to the configuration of another system (information source). The relation between this notion of information and more elaborate notions of information is given by the fact that the information-theoretic information is a minimal condition for any "elaborate information." In a physical theory it is sufficient to deal with this basic information-theoretic notion of information. This is very weak; it does not require us to consider information storage, thermodynamics, complex systems, meaning, or anything of the sort. In particular: (i) information can be lost dynamically (correlated state may become uncorrelated); (ii) we do not distinguish between "obtained" correlation and "accidental" correlation (if the pointer of the apparatus indicates the correct value of the spin, we say that the pointer has information about the spin, whether or not this is the outcome of a "well-thought" interaction); most important, (iii) any physical system may contain information about another physical system. For instance, if we have two spin-1/2 particles that have the same value of the spin in the same direction, we say that one has information about the other one. Thus "observer system" in this paper is any possible physical system (with more than one state). If there is any hope of understanding how a system may behave as observer without renouncing the postulate that all systems are equivalent, then the same kind of process—collapse—that happens between an electron and a CERN machine may also happen between an electron and another electron. Observers are not "physically special systems" in any sense. The relevance of information theory for understanding quantum physics has been advocated by Wheeler (1988, 1989, 1992).

Now, as Section 2.4 has shown, the fact that q has a value relative to O means that q is correlated with the pointer variable in O. Therefore, it means that the pointer variable in O has information about q, in the information-theoretic sense. This is the reason for choosing the expression "information" to denote the relational aspect of physical value ascriptions. Thus, the physical nature of the relation between S and O expressed in the fact that q has a value relative to O is captured by the fact that O has information (in the sense of information theory) about q. By "O has information about q" we mean "relative to O, q has a value" and also "relative to P, there is a certain correlation in the S and O states." Notice that this is, in a sense, a partial answer to the question formulated at the beginning of this section. First, it is a quantum mechanical answer, because P's information about the S–O system is probabilistic. Second, it is an answer that only shifts the problem by one step, because the information possessed by O is explained in terms of the information possessed by P. Thus, the notion of information I use has a double valence. On the one hand, I want to weaken all physical statements that we make: not "the spin is up," but "we have information that the spin is up"—which leaves the possibility open to the fact that someone else has different information. Thus, *information* indicates the usual ascription of values to quantities that founds physics, but emphasizes the relational aspect. On the other hand, this ascription can be described within the theory itself, as information-theoretic *information*, namely correlation. But such a description, in turn, is quantum mechanical and observer dependent, because a universal observer-independent description of the state of affairs of the world is fantasy.

Physics is the theory of the relative information that systems have about each other. This information exhausts everything we can say about the world.

At this point, all the main ideas and concepts have been formulated. In the next section, I consider a certain number of postulates expressed in terms of these concepts, and derive quantum mechanics from these postulates.

3. ON THE RECONSTRUCTION OF QUANTUM MECHANICS

3.1. Basic Concepts

Physics is concerned with relations between physical systems. In particular, it is concerned with the description that physical systems give about physical systems. Following Hypothesis 1, I reject any such fundamental distinctions as system/observer, quantum/classical system, physical system/consciousness. I assume that the world can be decomposed (possibly in a large number of ways) into a collection of systems, each of which can be equivalently considered as an *observing system* or as an *observed system*. A system (*observing system*) may have *information* about another system (*observed system*). Information is exchanged via physical interactions. The actual process through which information is collected and stored is not of particular interest here, but can be physically described in any specific instance.

Information is a discrete quantity: there is a minimum amount of information exchangeable (a single bit, or the information that distinguishes between just two alternatives). The process of acquisition of information (a measurement) can be described as a "question" that a system (observing system) asks another system (observed system). This anthropomorphic language is not meant to suggest any special feature or complexity of the systems considered. Since information is discrete, any process of acquisition of information can be decomposed into acquisitions of elementary bits of information. We refer to an elementary question that collects a single bit of information as a "yes/no question" and we denote these questions as Q_1, Q_2, ...

Any system S, viewed as an observed system, is characterized by a family of yes/no questions that can be meaningfully asked to it. These correspond to the physical variables of classical mechanics and to the observables of conventional quantum mechanics. We denote the set of these questions as

Щ$(S) = \{Q_i, i \in I\}$, where the index i belongs to a set I characteristic of S. The general kinematical features of S are representable as relations between the questions Q_i in Щ(S), that is, as structures over Щ(S). For instance, meaningful questions that can be asked of an electron are whether the particle is in a certain region of space, whether its spin along a certain direction is positive, and so on.

By asking a sequence of questions $(Q_1, Q_2, Q_3, \ldots)$ of S, an observer system O may compile a string

$$(e_1, e_2, e_3, \ldots) \tag{4}$$

where each e_i is either 0 or 1 ("no" or "yes") and represents the "answer" of the system to the question Q_i. (More precisely, the information that O has about S can be represented as a string.) It is of course a basic fact about nature that knowledge of a portion $(e_1, \ldots, e_n)$ of this string provides indications about the subsequent outcomes $(e_n, e_{n+1}, \ldots)$. It is in this sense that a string (4) contains the information that O has about S.

It is useful to distinguish between information contained in an arbitrary string (4) and *relevant* information. (Repeating the same question (experiment) and obtaining always the same outcome does not increase the information on S.) The *relevant information* (or simply *information*) that O has about S is defined as the nontrivial content of the (potentially infinite) string (4), that is, the part of (4) relevant for predicting future answers of possible future questions. The relevant information is the subset of the string (4) obtained by discarding the e_i that do not affect the outcomes of future questions.

The relation between the notions introduced and traditional notions used in quantum mechanics is rather transparent: A question is of course a version of a measurement. The idea that in quantum mechanics measurements can be reduced to yes/no measurements is well known. A yes/no measurement is represented by a projection operator onto a linear subset of the Hilbert space, or by the linear subset of the Hilbert space itself. Here this idea is not derived from the quantum mechanical formalism, but is justified in information-theoretic terms. The notions of observing system and observed system reflect traditional notions of observer and system; however, any sort of distinction between classical and quantum systems of the sort advocated by Bohr is rejected here. The set of questions that can be asked about a given system S, namely Щ(S), reflects the notion of the set of the observables. I recall that in algebraic approaches a system is characterized by the (algebraic) structure of the family of its observables.

On the other hand, there is a notion not mentioned here: the *state* of the system. The absence of this notion is the prime feature of the interpretation considered here. In place of the notion of state, which refers solely to the system, the notion of the information that a system has about another system has been introduced. I do not view the notion of information as "metaphysical," but as a concrete notion: I imagine a piece of paper on which outcomes of measurements on S are written, or hands of measuring apparatuses, or the memories of scientists, or a two-value variable which is "up" or "down" after an interaction.

For simplicity, in the following I focus only on systems that in conventional quantum mechanics can be described by means of finite-dimensional Hilbert spaces. This choice simplifies the mathematical treatment of the theory, avoiding continuum spectrum and other infinitary issues. I leave to future work the extension to continuum systems.

3.2. The Two Main Postulates

Postulate 1 (Limited information). There is a maximum amount of *relevant information* that can be extracted from a system.

The physical meaning of Postulate 1 is that it is possible to exhaust, or "give a complete description of the system," in a finite time. In other words, any future prediction that can be inferred about the system out of an infinite string (6) can also be inferred from a finite subset

$$s = [e_1, \ldots, e_N] \tag{5}$$

of (4), where N is a number that characterizes the system S. The finite string (5) represents the knowledge that O has about S.[3] One may say that any system S has a maximal "information capacity" N, where N, being an amount of information, is expressed in bits. This means that N bits of information will exhaust everything we can say about the system S. Thus, each system is characterized by a number N. In terms of traditional notions, we can view N as the smallest integer such that $N \geq \log_2 k$, where k is the dimension of the Hilbert space of the system S. Recall that the outcomes of the measurement of a complete set of commuting observables characterizes the state, and in a system described by a ($k = 2^N$)-dimensional Hilbert space such measurements distinguish one possible outcome out of 2^N alternative (the number of orthogonal basis vectors): this means that one gains information N on the system. Postulate 1 is confirmed by our experience about the world (within the assumption above, that we restrict to finite-dimensional Hilbert space systems; generalization to infinite systems should not be difficult).

Notice that Postulate 1 already adds Planck's constant to classical physics. Consider a classical system described by a variable q that takes continuous

[3]The string (5) is essentially the state of the system. The novelty here is not the fact that the state is defined as the response of the system to a set of yes/no experiments: this is the traditional reading of the state as a preparation procedure. The novelty is that this notion of state is relative to the observer that asked the questions.

values; for instance, the position of a particle. Classically, the amount of information we can gather about it is infinite: we can locate its state in the system's phase space with arbitrary precision. Quantum mechanically, this infinite localization is impossible because of Postulate 1. Thus, maximum available information can localize the state only within a finite region of the phase space. Since the dimensions of the classical phase space of any system are $[L^2 T^{-1} M]^n$, this implies that there is a universal constant with dimension $[L^2 T^{-1} M]$ that determines the minimal localizability of objects in phase space. This constant is of course Planck's constant. Thus we can view Planck's constant just as the transformation coefficient between physical units (position $\times$ momentum) and information-theoretic units (bits).

What happens if, after having asked the N questions such that the maximal information about S has been gathered, the system O asks a further question $Q = Q_{N+1}$? I introduce the second postulate.

Postulate 2 (Unlimited questions). It is always possible to acquire *new* information about a system.

If, after having gathered the maximal information about S, the system O asks a further question Q of the observed system S, there are two extreme possibilities: either the question Q is fully determined by previous questions, or not. In the first case, no new information is gained. However, the second postulate assures us that there is always a way to acquire new information. This postulate implies therefore that the sequence of responses we obtain from observing a system cannot be fully deterministic.

The motivation for the second postulate is fully experimental. We know that all quantum systems (and all systems are quantum systems) have the property that even if we know their quantum state $|\psi\rangle$ exactly, we can still "learn" something new about them by performing a measurement of a quantity O such that $|\psi\rangle$ is not an eigenstate of O. This is an *experimental* result about the world, coded in quantum mechanics. Postulate 2 expresses this result. Postulate 2 is true to the extent that Planck's constant is different from zero: in other words, for a macroscopic system, getting to questions that increase our knowledge of the system after having reached the maximum of our information implies measurements with extremely high sensitivity.

Since the amount of information that O can have about S is limited by Postulate 1, when new information is acquired, part of the old relevant information must become irrelevant. In particular, if a new question Q (not determined by the previous information gathered), is asked, then O should lose (at least) one bit of the previous information. Thus, after asking the question Q, new information is available, but the total amount of information about the system does not exceed N bits.

Rather surprisingly, those two postulates are (almost) sufficient to reconstruct the full formalism of quantum mechanics. Namely, one may assert that the physical content of the general formalism of quantum mechanics is (almost) nothing but a sequence of consequences of two physical facts expressed in Postulates 1 and 2. This is illustrated in the next section.

3.3. Reconstruction of the Formalism, and the Third Postulate

In this section, I discuss the possibility of deriving the full formalism of quantum mechanics simply from the two simple physical assertions contained in Postulates 1 and 2. This section is rather technical, and the uninterested reader may skip it and jump to Section 3.4. The technical machinery to be employed has been developed (in a somewhat different spirit) in quantum logic analyses. See, for example, Beltrametti and Cassinelli (1981). As I mentioned in the introduction, this reconstruction attempt is not fully successful. In fact I will be forced to introduce a third postulate (besides various relative minor assumptions). I will speculate on the possibility of giving this postulate a simple physical meaning, but I do not have any clear result. This difficulty reflects the parallel difficulties in the attempts at quantum logic reconstruction.

Let me begin by analyzing the consequences of the first postulate. The number of questions in Щ(S) can be much larger than N. Some of these questions may not be independent. In particular, one may find (experimentally) that they can be related by implication ($Q_1 \Rightarrow Q_2$), union ($Q_3 = Q_1 \vee Q_2$), and intersection ($Q_3 = Q_1 \wedge Q_2$), that we can define an always-false (Q_0) and an always-true question (Q_∞), the negation of a question (not-Q), and a notion of orthogonality as follows: If $Q_1 \Rightarrow$ not-Q_2, then Q_1 and Q_2 are orthogonal (we indicate this as $Q_1 \perp Q_2$). Equipped with these structures, and under the (nontrivial) additional assumption that $\vee$ and $\wedge$ are defined for every pair of questions, Щ(S) is an orthomodular lattice (Beltrametti and Cassinelli, 1981; Hughes, 1989).

If there is a maximal amount of information that can be extracted from the system, we may assume that one can select in Щ(S) an ensemble of N questions Q_i, which we denote as $ч = \{Q_i, i = 1, \ldots, N\}$, that are independent from each other. We do not assume there is anything canonical in this choice, so that there may be many distinct families ч, б, ъ, . . . of N independent questions in Щ(S). If a system O asks the N questions in the family ч of a system S, then the answers obtained can be represented as a string that we may denote as

$$s_ч = [e_1, \ldots, e_N]_ч \qquad (6)$$

The string $s_ч$ represents the information that O has about S as a result of the interaction that allowed it to ask the questions in ч. The string $s_ч$ can take $2^N = K$ values; we denote these values as $s_ч^{(1)}, s_ч^{(2)}, \ldots, s_ч^{(K)}$. Thus

$$s_ч^{(1)} = [0, 0, \ldots, 0]_ч, \ldots, \quad s_ч^{(K)} = [1, 1, \ldots, 1]_ч \tag{7}$$

Since the 2^N possible outcomes $s_ч^{(1)}, s_ч^{(2)}, \ldots, s_ч^{(K)}$ of the N yes/no questions are (by construction) mutually exclusive, we can define 2^N new questions $Q_ч^{(1)}, \ldots, Q_ч^{(K)}$ such that the "yes" answer to $Q_ч^{(i)}$ corresponds to the string of answers $s_ч^{(i)}$:

$$\begin{aligned} Q_ч^{(1)} &= \text{not-}Q_1 \wedge \text{not-}Q_2 \wedge \cdots \wedge \text{not-}Q_N \\ Q_ч^{(2)} &= \text{not-}Q_1 \wedge \text{not-}Q_2 \wedge \cdots \wedge Q_N \\ &\ldots \\ Q_ч^{(k)} &= Q_1 \wedge Q_2 \wedge \cdots Q_N \end{aligned} \tag{8}$$

We refer to questions of this kind as "complete questions." By taking all possible unions of sets of complete questions $Q_ч^{(i)}$ (of the same family ч), we can construct a Boolean algebra that has $Q_ч^{(i)}$ as atoms.

Alternatively, the observer O could use a *different* family of N independent yes–no questions in order to gather information about S. Let us denote this other set as б. Then, he will still have a maximal amount of relevant information about S formed by an N-bit string $s_б = [e_1, \ldots, e_N]_б$. Thus, O can give different kinds of descriptions of S, by asking different questions. Correspondingly, we denote as $s_б^{(1)}, \ldots, s_б^{(K)}$ the 2^N values that $s_б$ can take, and we consider the corresponding complete questions $Q_б^{(1)}, \ldots, Q_б^{(K)}$ and the Boolean algebra they generate. Thus, it follows from the first postulate that the set of the questions Щ(S) that can be asked of a system S has a natural structure of an orthomodular lattice containing subsets that form Boolean algebras. This is precisely the algebraic structure formed by the family of the linear subsets of a Hilbert space, which represent the yes/no measurements in ordinary quantum mechanics (Jauch, 1968; Finkelstein, 1969; Piron, 1972; Beltrametti and Cassinelli, 1981).

The next question is the extent to which the information (6) about the set of questions ч determines the outcome of an additional question Q. There are two extreme possibilities: that Q is fully determined by (6), or that it is fully independent, namely that the probability of getting a "yes" answer is 1/2. In addition, there is a range of intermediate possibilities: The outcome of Q may be determined probabilistically by $s_ч$. The second postulate states explicitly that there are questions that are nondetermined. We may define, in general, as $p(Q, Q_ч^{(i)})$ the probability that a "yes" answer to Q will follow the string $s_c^{(i)}$. Given two complete families of information s_c and s_b, we can then consider the probabilities[4]

$$p^{ij} = p(Q_b^{(i)}, Q_b^{(j)}) \tag{9}$$

From the way it is defined, the $2^N \times 2^N$ matrix p^{ij} cannot be fully arbitrary. First, we must have

$$0 \leq p^{ij} \leq 1 \tag{10}$$

Then, if the information $s_c^{(j)}$ is available about the system, one and only one of the outcomes $s_b^{(i)}$ may result. Therefore

$$\sum_i p^{ij} = 1 \tag{11}$$

We also assume that $p(Q_b^{(i)}, Q_c^{(j)}) = p(Q_c^{(j)}, Q^{(i)})$ (this is a new assumption! there is a relation with time reversal, but I leave it here as an unjustified assumption at this stage), from which we must have

$$\sum_j p^{ij} = 1 \tag{12}$$

The conditions (10)–(12) are strong constraints on the matrix p^{ij}. They are satisfied if

$$p_j = |U^{ij}|^2 \tag{13}$$

where U is a unitary matrix, and p^{ij} can always be written in this form for some unitary matrix U (which, however, is not fully determined by p^{ij}).

In order to take into account questions which in the Boolean algebra are generated by a family s_c, for instance,

$$Q_c^{(jk)} = Q_c^{(j)} \vee Q_c^{(k)} \tag{14}$$

we cannot consider probabilities of the form $p(Q_b^{(i)}, Q_c^{(jk)})$ because a "yes" answer to $Q_c^{(jk)}$ is less than the maximum amount of relevant information. But we may, for instance, consider probabilities of the form

$$p^{i(jk)i} = p(Q_b^{(i)}, Q_c^{(jk)} Q_b^{(i)}) \tag{15}$$

defined as the probability that a "yes" answer to $Q_b^{(i)}$ will follow a "yes" answer to $Q_b^{(i)}$ (N bits of information) and a subsequent "yes" answer to

[4] I do not wish to enter here the debate on the meaning of probability in quantum mechanics. I think that the shift of perspective I am suggesting is meaningful in the framework of an "objective" definition of probability, tied to the notion of repeated measurements, as well as in the context of "subjective" probability, or any variant of this, if one does not accept Jayne's criticisms of the last. The novelty of the proposal regards the notion of state, not the notion of probability.

$Q_{\text{ч}}^{(jk)}$ ($N - 1$ bits of information). As is well known, we have (experimentally!) that

$$p^{i(jk)i} \neq p(Q_{\text{б}}^{(i)}, Q_{\text{ч}}^{(j)})p(Q_{\text{ч}}^{(j)}, Q_{\text{б}}^{(i)}) + p(Q_{\text{б}}^{(i)}, Q_{\text{ч}}^{(k)})p(Q_{\text{ч}}^{(k)}, Q_{\text{б}}^{(i)})$$
$$= (p^{ij})^2 + (p^{ik})^2 \tag{16}$$

Accordingly, we can determine the missing phases of U in (21) by means of

$$p^{i(jk)i} = |U^{ij}U^{ji} + U^{ik}U^{ki}|^2 \tag{17}$$

It would be extremely interesting to study the constraints that the probabilistic nature of the quantities p implies, and to investigate the extent to which the structure of quantum mechanics can be derived in full from these constraints. One could conjecture that equations (13)–(17) could be derived solely by the properties of conditional probabilities—or find exactly the weakest formulation of the superposition principle directly in terms of probabilities: this would be a strong result. Even more interesting would be to investigate the extent to which the already noticed consistency between different observers' descriptions, which I believe characterizes quantum mechanics so marvelously, could be taken as the missing input for reconstructing the full formalism. I have a suspicion this could work, but have no definite result. Here, I content myself with the much more modest step of introducing a third postulate. For strictly related attempts to reconstruct the quantum mechanical formalism from the algebraic structure of the measurement outcomes, see Mackey (1963), Maczinski (1967), Finkelstein (1969), Jauch (1968), and Piron (1972).

Postulate 3 (Superposition principle). If ч and б define two complete families of questions, then the unitary matrix $U_{\text{чб}}$ in

$$p(Q_{\text{ч}}^{(i)}, Q_{\text{б}}^{(j)}) = |U_{\text{чб}}^{ij}|^2 \tag{18}$$

can be chosen in such a way that for every ч, б, and р, we have $U_{\text{чр}} = U_{\text{чб}}U_{\text{бр}}$ and the effect of composite questions is given by (17). It follows that we may consider any question as a vector in a complex Hilbert space, fix a basis $|Q_{\text{ч}}^{(i)}\rangle$ in this space, and represent any other question $|Q_{\text{б}}^{(j)}\rangle$ as a linear combination of these:

$$|Q_{\text{б}}^{(j)}\rangle = \sum_i U_{\text{бч}}^{ij} |Q_{\text{ч}}^{(i)}\rangle \tag{19}$$

The matrices $U_{\text{бч}}^{ij}$ could then be interpreted as generating a unitary change of basis from the $|Q_{\text{ч}}^{(i)}\rangle$ to the $|Q_{\text{б}}^{(j)}\rangle$ basis. Recall now the conventional quantum mechanical probability rule: if $|v^{(i)}\rangle$ are a set of basis vectors and $|w^{(j)}\rangle$ a second set of basis vectors related to the first ones by

$$|w^j\rangle = \sum_i U^{ji} |v^i\rangle \tag{20}$$

then the probability of measuring the state $|w^j\rangle$ if the system is in the state $|v^i\rangle$ is

$$p^{ij} = |\langle v^i | w^j\rangle|^2 \tag{21}$$

(20) and (21) yield $p^{ij} = |U^{ij}|^2$, which is equation (18). Therefore the conventional formalism of quantum mechanics and its probability rules follow. The set Щ(S) has the structure of a set of linear subspaces in the Hilbert space. For any yes/no question Q_i, let L_i be the corresponding linear subset of H. The relations $\{\Rightarrow, \vee, \wedge, \text{not}, \perp\}$ between questions Q_i correspond to the relations {inclusion, orthogonal sum, intersection, orthogonal-complement, orthogonality} between the corresponding linear subspaces L_i. The kinematics of quantum mechanics can therefore be reconstructed from the three postulates given. Here, however, notice that the Hilbert space on which state vectors live is not attached to a system, but to *two* systems, namely to a system–observer pair.

The inclusion of dynamics in the above scheme is then conventional. We simply notice that two questions can be considered as different questions if defined by the same operations but asked at different moments of time. Thus, any question Q can be labeled by the time variable t, indicating the time at which it is asked: we denote as $t \to Q(t)$ the one-parameter family of questions defined by the same procedure performed at different times. As we have seen, the set Щ(S) has the structure of a set of linear subspaces in the Hilbert space. Assuming that time evolution is a symmetry in the theory, then the set of all the questions at time t_2 must be isomorphic to the set of all the questions at time t_1. Therefore the corresponding family of linear subspaces must have the same structure; therefore there should be a unitary transformation $U(t_2 - t_1)$ such that

$$Q(t_2) = U(t_2 - t_1)Q(t_1)U^{-1}(t_2 - t_1) \tag{22}$$

By conventional arguments, we then have that these unitary matrices form an Abelian group and that $U(t_2 - t_1) = \exp\{-i(t_2 - t_1)H\}$, where H is a self-adjoint operator on the Hilbert space, the Hamiltonian. The Schrödinger equation follows immediately. Thus, the time evolution, too, can be viewed as a structure on the set of the questions that can be asked of a system, or, more precisely, a structure on the family of all the questions that can be asked of the system at all times.

3.4. The Observer Observed

We now have the full formal machinery of quantum mechanics, but with some interpretative novelty: There is no "state of the system" in this framework. At this point, I can return to the issue of the relation between information of observers. The important point in this regard is that the information possessed by different observers cannot be compared directly. This is a delicate but crucial point of the entire construction. A statement about the information possessed by O is a statement about the physical state of O; the observer O is a regular system on the same ground as any other system; thus, we must discuss his state in physical terms. However, since *there is no absolute meaning to the state of a system*, any statement regarding the state of O is to be referred to some other system observing O. The notion of absolute *state* of a system, and thus *a fortiori* absolute state of an observer, is not defined. Therefore, the fact that an observer has information about a system is not an absolute fact: it is something that can be observed by an observer. A second observer P can have information about the fact that O has information about S. But any acquisition of information implies a physical interaction. P can get new information about the information that O has about S only by physically interacting with the O–S system.

I believe that a common mistake in analyzing measurement issues in quantum mechanics is to forget that precisely as an observer can acquire information about a system only by physically interacting with it, in the same fashion two observers can compare their information only by physically interacting with each other. This means that there is no way to compare "the information possessed by O" with "the information possessed by P" without considering a physical interaction between the two. Information, like any other property of a system, is a fully relational notion.

How can a system P have information about the fact that O has information about S? The observer P considers the coupled S–O system. Thus, she may ask questions of S, or of O, or of the two together. In particular she may ask questions concerning the information that O has about S. Information is simply a property of some degrees of freedom in O being correlated with some property of S. A question about the information possessed by O is in no way different from any other physical question. As far as P is concerned, knowing the physics of the S–O system means knowing the set Щ(S–O) of the meaningful questions that can be asked of the S–O system and its full structure. In particular, it also means knowing how answers at time t_1 determine answers at time t_2.

At the risk of repeating what was presented in Section 2.5, let me describe the information that O has about S, as it is contained in the information that P possesses. This exercise will also show how the question formalism may work. All questions below are posed by P. A meaningful question to ask S is whether $q = 1$ is true. Denote this question as Q_1. Notice that the fact that this is a meaningful question to ask S is not relational (is not contingent), and thus both O and P can ask this same question of S. A meaningful (complete) question about O is whether his measuring apparatus is ready to (and going to) ask S the question of whether $q = 1$ is true. Denote this question (that P can ask of O) as Q_{Ready}. Another meaningful question is whether the hand of his measuring apparatus is on "$O1$." Denote this question as Q_{O1}. Similarly for Q_{O2}. Each of these questions can be asked at any time t.

$$
\begin{aligned}
&Q_{\text{Ready}}(t): && O \text{ is going to measure whether } q = 1 \text{ on } S \\
&Q_1(t): && q = 1 \\
&Q_2(t): && q = 2 \\
&Q_{O1}(t): && O \text{ has the information that } q = 1 \\
&Q_{O2}(t): && O \text{ has the information that } q = 2
\end{aligned}
\tag{23}
$$

We also consider a complete question $Q_{\text{Mix}}(t)$ to S such that

$$p(Q_1(t), Q_{\text{Mix}}(t)) = p(Q_2(t), Q_{\text{Mix}}(t)) = 1/2 \tag{24}$$

and define

$$Q_{\text{Ready,Mix}}(t_1) = Q_{\text{Ready}}(t) \wedge Q_{\text{Mix}}(t) \tag{25}$$

Knowing the dynamics of the coupled S–O system, P can work out the possible outcomes of questions asked at time t_2, given a yes answer to $Q_{\text{Ready,Mix}}(t_1)$. Assuming standard Hilbert-space Hamiltonian dynamics and assuming that the coupling Hamiltonian produces a good measurement, P will compute the probabilities

$$p(Q_1(t_2), Q_{\text{Ready,Mix}}(t_1)) = 1/2 \tag{26a}$$

$$p(Q_2(t_2), Q_{\text{Ready,Mix}}(t_1)) = 1/2 \tag{26b}$$

$$p(Q_{O1}(t_2), Q_{\text{Ready,Mix}}(t_1)) = 1/2 \tag{26c}$$

$$p(Q_{O2}(t_2), Q_{\text{Ready,Mix}}(t_1)) = 1/2 \tag{26d}$$

Thus, she has no information whether $q = 1$ or $q = 2$ at time t_2, nor information about what the hand of the O apparatus indicates.

However, she may consider asking the questions whether O knows about q or not. By this we mean whether or not the pointer is on the right position. Let us denote this question as $Q_{\text{"}O\text{-knows"}}$. This is the conjunction of two coupled questions:

$$Q_{\text{"}O\text{-knows"}} = [Q_1 \wedge Q_{O1}] \vee [Q_2 \wedge Q_{O2}] \tag{27}$$

and corresponds to the operator M described in Section 2.5. Note that Q_1, Q_{O1}, Q_2, and Q_{O2} are compatible questions, so we can consider questions in the Boolean algebra they generate. The dynamics gives

$$p(Q_{\text{"}O\text{-knows"}}(t_2), Q_{\text{Ready,Mix}}(t_1)) = 1 \tag{28}$$

Thus, P has information that O has information about S [equation (28)]. This of course is not in contradiction with the fact that she (P) has no information about S [equations (26a), (26b)], nor has information about which specific information O has about S [equations (26c), (26d)]. Thus the notion "a system O has information about a system S" is a physical notion that can be studied experimentally (by a third observer) in the same way as any other physical property of a system. It corresponds to the fact that relevant variables in systems S and O are correlated.

Now, the question "Do observers O and P get the *same* answers out of a system S?" is a meaningless question, because it is a question about the *absolute state* of O and P. What is meaningful is to rephrase this question in terms of some observer. For instance, we could ask this question in terms of the information possessed by a further observer, or, alternatively, by P herself. Consider this last case. At time t_1, O gets information about S. Observer P has information about the initial state, and therefore has the information that the measurement has been performed. The meaning of this is that she knows that the states of the O–S systems are correlated, or more precisely she knows that if at a later time t_3 she asks a question of S concerning property A, and a question of O concerning his knowledge about A (or, equivalently, concerning the position of a pointer), she will get consistent results.

From the dynamical point of view, notice that knowledge of the structure of the family of questions $Ш(S)$ that can be asked of S implies the knowledge of the dynamics of S [because $Ш(S)$ includes all Heisenberg observables at all times]. In Hilbert space terms, this means knowing the Hamiltonian of the evolution of the observed system. If P knows the dynamics of the O–S system, she knows the two Hamiltonians of O and S *and* the interaction Hamiltonian. The interaction Hamiltonian cannot be vanishing because a measurement (O measuring S) implies an interaction: this is the only way in which a correlation can be dynamically established. From the point of view of P, the measurement is therefore a fully unitary evolution, which is determined by the interaction Hamiltonian between O and S. The interaction is a measurement if it brings the states to a correlated configuration. On the other hand, O gives a dynamical description of S alone. Therefore he can only use the S Hamiltonian. Since between times t_1 and t_2 the evolution of S is affected by its interaction with O, the description of the unitary evolution of S given by O breaks down. The unitary evolution does not break down for mysterious physical quantum jumps, due to unknown effects, but simply because O is not giving a full dynamical description of the interaction. O cannot have a full description of the interaction of S with himself (O), because his information is correlation, and there is no meaning in being correlated with oneself.

The reader may be convinced that even if we take into account several observers observing each other, there is no way in which contradictions may develop, provided that one does not violate the following two rules:

(i) There is no meaning to the state of a system or the information that a system has, except within the information of a further observer.

(ii) There is no way a system P may get information about a system O without physically interacting with it, and therefore without breaking down (at the time of the interaction) the unitary evolution description from any observer not including both S and O (*and* their interaction Hamiltonian!) in its Hilbert space description of the events.

For instance, there is no way two observers P and O can get information about a system S independently from each other: one of two (say O) will have to obtain the information first. In doing so, he will interact with S at a certain time t. This interaction implies that there is a nonvanishing interaction Hamiltonian between S and O. If P asks a question of O at a later time t', she will either have to consider the interacting correlated O–S system or realize that the unitary evolution of the O dynamics has broken down, due to some physical interaction she was not taking into account.

Finally, one can quantitatively study the relation between correlation of quantum states and amount of information. I will not pursue this direction here, but I present two comments in this regard. We can say in general that the property "$O1$" of the system O contains one bit of information about the property "1" of the system S any time the S–O system has answered a question Q such that

$$p(Q, Q_{\text{“}O\text{-knows-1”}}) = 1 \tag{29}$$

where

$$Q_{\text{“}O\text{-knows-1”}} = [Q_1 \wedge Q_{O1}] \vee [\text{not-}Q_1 \wedge \text{not-}Q_{O1}] \tag{30}$$

and so on. There is an alternative (independent) characterization of amount of information, as the amount of entanglement between S and O. Using the Hilbert space formalism, we may consider the state $|\Psi\rangle$ in the tensor product Hilbert space and the corresponding pure state density matrix $\rho = |\Psi\rangle\langle\Psi|$. If we denote the traces in the two factor Hilbert spaces by Tr_S and Tr_O, then it is easy to see that for a nonentangled state $|\Psi\rangle = |\psi\rangle_S \otimes |\phi\rangle_O$ we have

$$\text{Tr}_O(\text{Tr}_S\rho)^2 = 1 \tag{31}$$

while for an entangled state of the form

$$|\Psi\rangle = \frac{1}{\sqrt{n}} \sum_{i=1,\ldots,n} |\psi_i\rangle_S \otimes |\phi_i\rangle_O \tag{32}$$

we have

$$\mathrm{Tr}_O(\mathrm{Tr}_S\rho)^2 = 1/n \qquad (33)$$

Thus, we may consider the amount of "entangling information" defined as

$$N = -\ln \mathrm{Tr}_O(\mathrm{Tr}_S\rho)^2 \qquad (34)$$

On the possibility of defining mutual information from quantum states, see Halliwell (1994).

4. CRITIQUE OF THE CONCEPT OF STATE

4.1. "Any Observation Requires an Observer:" Summary of the Ideas Presented

Let me summarize the path covered. I started from the distinction between observer and observed system. I assumed (Hypothesis 1) that all systems are equivalent, in the sense that any observer can be described by the same physics as any other system. In particular, I assumed that an observer that measures a system can be described by quantum mechanics. I analyzed a fixed physical sequence of events **E** from two different points of observation, one that of the observer and one that of a third system, external to the measurement. I concluded that two observers give different accounts of the same physical set of events (main observation).

Rather than backtracking from this observation and giving up the commitment to the belief that all observers are equivalent, I decided to take this experimental fact at its face value, and consider it as a starting point for understanding the world. *If different observers give different descriptions of the state of the same system, this means that the notion of state is observer dependent.* I took this deduction seriously, and considered a conceptual scheme in which the notion of absolute observer-independent state of a system is replaced by the notion of information about a system that a physical system may possess.

I considered three postulates that this information must satisfy, which summarize present experimental evidence about the world. The first limits the amount of relevant information that a system can have; the second summarizes the novelty revealed by the experiments from which quantum mechanics derives by asserting that whatever the information we have about a system, we can always get new information. The third limits the structure of the set of questions; this third postulate can probably be sharpened. Out of these postulates, the conventional Hilbert space formalism of quantum mechanics and the corresponding rules for calculating probabilities (and therefore any other equivalent formalism) can be rederived.

A physical system is characterized by the structure on the set Щ(S) of questions that can be asked of the system. This set has the structure of the non-Boolean algebra of a family of linear subspaces of a complex k-dimensional Hilbert space. The information about S that any observer O can possess can be represented as a string s, containing an amount of information N.

I investigated the meaning of this "information" out of which the theory is constructed. I showed that the fact that a variable in a system O has information about a variable in a system S means that the variables of S and O are correlated, meaning that a third observer P has information about the coupled S–O system that allows her to predict correlated outcomes between questions to S and questions to O. Thus correlation has no absolute meaning, because states have no absolute meaning, and must be interpreted as the content of the information that a third system has about the S–O couple.

Finally, since we take quantum mechanics as a complete description of the world at the present level of experimental knowledge (Hypothesis 2), we are forced to accept the result that there is no "objective," or more precisely "observer-independent," meaning to the ascription of a property to a system. Thus, the properties of the systems are to be described by an interrelated net of observations and information collected from observations. Any complex situation can be described "*in toto*" by a further additional observer, and the interrelation is consistent. However, such an "*in toto*" description is deficient in two directions: upward, because an even more general observer is needed to describe the global observer itself, and, more importantly, downward, because the "*in toto*" observer knows the content of the information that the single component systems possess about each other only probabilistically.

There is no way to "exit" from the observer-observed global system. *Any observation requires an observer* [the expression is taken from Maturana and Varela (1980)]. In other words, I suggest that it is a matter of natural science whether or not the descriptions that different observers give of the same ensemble of events is universal or not:

Quantum mechanics is the theoretical formalization of the experimental discovery that the descriptions that different observers give of the same events are not universal.

The concept that quantum mechanics forces us to give up is the concept of a description of a system independent of the observer providing such a description; that is, the concept of absolute state of a system. The structure of the classical scientific description of the world in terms of *systems* that are in certain *states* is perhaps incorrect, and inappropriate to describe the world beyond the $\hbar \to 0$ limit.

It is perhaps worthwhile to emphasize that those considerations do not follow from the theory of quantum mechanics: they follow from a collection of experiments on the atomic world.

There are several aspects of the point of view discussed here that require further development: (i) The reconstruction of Section 3 can certainly be sharpened, and Postulate 3 should be better understood. (ii) The quantitative relation between amount of information and correlation between states which was hinted at at the end of Section 3.4 can be studied. (iii) I believe it would be interesting to reconsider EPR-like issues in the light of the considerations made here; notice that there is no meaning in comparing the outcome of two spatially separated measurements unless there is a physical interaction between the observers. I conclude with a brief discussion of the relation between the view presented here and some of the popular views of quantum mechanics.

4.2. Relation with Other Interpretations

I follow Butterfield (1995) to organize current strategies on the quantum puzzle. The first strategy (Dynamics) is to reject the quantum postulate that an isolated system evolves according to the linear Schrödinger equation and consider additional mechanisms that modify this evolution, in a sense physically producing the wave function collapse. Examples are the interpretations in which the measurement process is replaced by some hypothetical process that violates the linear Schrödinger equation (Ghirardi *et al.*, 1986; Penrose, 1989). Those interpretations are radically different from the present approach, since they violate Hypothesis 2. My effort here is not to modify quantum mechanics to make it consistent with my view of the world, but to modify my view of the world to make it consistent with quantum mechanics.

The second and third strategies maintain the idea that probabilistic expectations of values of any isolated physical system are given by the linear Schrödinger evolution. They must face the problem of reconciling probabilities expressed by the state at time t_2 in equation (2) [$q = 1$ with probability 1/2 and $q = 2$ with probability 1/2] with the assertion that the observer O assigned the value $q = 1$ to the variable q at the same time t_2. As Butterfield emphasizes, if this value assignment coexists with the probability distribution expressed by (2), then the eigenstate–eigenvalue link must be in some sense weakened, and the possibility of assigning values to variables in addition to the eigenstate case (extra values) allowed. The second and the third strategies in Butterfield's classification differ about whether these extra values are "wholly a matter of physics" (Physics Values), or are "somehow mental or perspectival" (Perspectival Values). In the first case, the assignment is (in every sense) observer independent. In the second case, it is (in some sense) observer dependent.

A prime example of the second (Physics Values) strategy is Bohr's, or the Copenhagen, interpretation—at least in one possible reading. Bohr assumes a classical world. In Bohr's view, this classical world is physically distinct from the microsystems described by quantum mechanics, and it is precisely the classical nature of the apparatus that gives measurement interactions a special status (Bohr, 1949) [for a clear discussion of this point, see Landau and Lifshitz (1977)].

From the point of view developed here, we can fix once and for all a privileged system S_o as "The Observer" (capitalized). This system S_o can be formed, for instance, by all the macroscopic objects around us. In this way we recover Bohr's view entirely. The quantum mechanical "state" of a system S is then the information that the privileged system S_0 has about S. Bohr's choice is simply the assumption of a large (consistent) set of systems (the classical systems) as privileged observers. This is fully consistent with the view proposed here.[5] By taking Bohr's step, one becomes blind to the net of interrelations that are at the foundation of the theory, and, more importantly, puzzled about the fact that the physical theory treats one system, S_0—the classical world—in a way which is physically different from the other systems. The disturbing aspect of Bohr's view is the inapplicability of quantum theory to macrophysics. This disturbing aspect vanishes, I believe, in the light of the discussion in this paper.

Therefore, the considerations in this paper do not suggest any modification to the conventional *use* of quantum mechanics: there is nothing incorrect in fixing the preferred observer S_0 once and for all. Thus, acceptance of the point of view suggested here implies continuing to use quantum mechanics precisely as it is currently used. On the other hand, this point of view (I hope) could bring some clarity about the physical significance of the strange theoretical procedure adopted in Bohr's quantum mechanics: treating a portion of the world in a different way than the rest of the world. This different treatment is, I believe, the origin of the widespread unease with quantum mechanics.

The strident aspect of Bohr's quantum mechanics is cleanly characterized by von Neumann's introduction of the "projection postulate," according to which systems have two different kinds of evolutions: the unitary and deterministic Schrödinger evolution, and the instantaneous, probabilistic measure-

[5] A separate problem is why the observing system chosen—S_0, the macroscopic world—admits, in turn, a description in which expectation probabilities evolve classically, namely are virtually always concentrated on values 0 and 1, and interference terms are invisible. It is to *this* question that the physical decoherence mechanism (Joos and Zeh, 1985; Zurek, 1981) provides an answer. Namely, *after* having an answer on what determines extra values ascriptions (the observer–observed structure, in the view proposed here), the physical decoherence mechanism helps explain why those ascriptions are consistent with classical physics in macroscopic systems.

ment collapse (von Neumann, 1932). According to the point of view described here, the Schrödinger unitary evolution of the system S breaks down simply because the system interacts with something which is not taken into account by the evolution equations. Unitary evolution requires the system to be isolated, which is exactly what ceases to be true during the measurement, because of the interaction with the observer. If we include the observer into the system, then the evolution is still unitary, but we are now dealing with the description of a different observer. As suggested by Ashtekar (1993), the point of view presented here can then be characterized by a fundamental assumption prohibiting an observer to be able to give a full description of "itself." In this respect, these ideas are related to earlier suggestions that quantum mechanics is a theory that necessarily excludes the observer (Peres and Zurek, 1982; Roessler, 1987; Finkelstein, 1988; Primas, 1990). A recent result in this regard is a general theorem proven by (Breuer, 1994) according to which no system (quantum *or* classical) can perform a complete self-measurement. Breuer showed on general grounds that no system O can possess internal records capable of distinguishing between quantum mechanical states in which the system itself has different correlations with another system S. If measuring the state of S implies the creation of correlations between S and O, then it follows that there is a residual irreducible indeterminacy in any information about a system. We believe that the relation between the point of view presented here and Breuer's result deserves to be explored.

Other views within Butterfield's second strategy (Physical Values) are Bohm's hidden-variables theory, which violates Hypothesis 2 (completeness), and modal interpretations, which deny the collapse but assume the existence of values of physical quantities. Of these, I am familiar with van Fraassen (1991), or the idea of actualization of potentialities in Shimony (1969) and Fleming (1992). The assumed values must be consistent with the standard theory's predictions, be probabilistically determined by a unitary evolving wave function, but are not constrained by the eigenstate–eigenvalue link. One may doubt that these acrobatics could work (Bacciagaluppi, 1995; Bacciagaluppi and Hemmo, n.d.). I am very sympathetic with the key idea that the object of quantum mechanics is a set of values of quantities and their distribution. Here, I have assumed value assignment as in these interpretations, but with two crucial differences. First this value assignment is observer dependent. Second, it need not be consistent with a fully unitary Schrödinger evolution, because the evolution is not unitary when the observed system interacts with the observer. Namely, there is collapse in each observer-dependent evolution of expected probabilities. Clearly, these two differences allow me to assign values to physical quantities without any of the consistency worries that plague modal interpretations. The point is that the break of the eigenstate–eigenvalue link is bypassed by the fact that the eigenvalue refers to one observer and the state to a different observer. For a fixed observer, the eigenstate–eigenvalue link is maintained. Consistency should only be recovered between different observers, but—this is a key point—consistency is only quantum mechanical—probabilistic—as discussed in detail in Section 3.4. Actuality is observer dependent. The fact that the values of physical quantities are relational and their consistency is only probabilistically required circumvents the potential difficulties of the modal interpretations.

A class of interpretations of quantum mechanics that Butterfield does not include in his classification, but which are presently very popular among physicists, is given by the consistent histories interpretation (Griffiths, 1984; Omnès, 1988) and the related coherent—or decoherent—histories interpretation (Gell-Mann and Hartle, 1990). These interpretations reduce the description of a system to the prediction of temporal sequences of values of physical variables. The key novelties are three : (i) probabilities are assigned to sequences of values, as opposed to single values; (ii) only certain sequences can be considered; (iii) probability is interpreted as probability of the given sequence of values within a chosen family of sequences. The restriction (ii) incorporates the quantum mechanical prohibition of giving value, say, to position and momentum at the same time. More precisely, in combination with (ii) it excludes all the instances in which observable interference effects make probability assignments inconsistent. In a sense, the histories formulations represent a sophisticated implementation of the program of discovering a minimum consistent value attribution scheme. The price paid for consistency is that a single value attribution is meaningless: indeed, whether or not a variable has a value may very well depend on whether we are asking or not if at a later time another variable has a value! In Section 2.2, I argued that within the history interpretations different observers *do* make different statements about the same events if they choose to work with different (consistent *families* of) histories. The question relevant to a comparison of this approach with the present work is then: What is the relation between probabilities computed by using one family or another? The situation is even more serious if we consider factual statements about nature, as opposed to probabilistic predictions. In the histories approaches two people may choose to work with two different families of consistent histories, and therefore give quite unrelated (and possibly contradictory) accounts of the same events.[6]

[6]Consider the following example: suppose we want to make statements about the past evolution of a system (perhaps the Earth) within a history formulation. I choose a certain consistent set of histories S appropriate to the system and compute their probabilities. Suppose for simplicity that I obtain that all the alternative histories have negligible probability, except the history A, which has probability close to 1 (perhaps in this history dinosaurs were killed by a meteorite). Then, I would like to say that A is exactly what has happened. You, the reader, choose a *different* consistent set of histories S'. Suppose that you compute that all the alternative

See also Objection 8 in Section 2.2, and Kent and Dowker (1994). Here, I maintain that the observer does not choose the family of histories in terms of which she describes the world out of pure thought. The choice is dictated by the physical interaction between the observer and the observed world (I realize that this is taboo in the histories world). The histories interpretations are not inconsistent with the analysis developed here. What I tried to add here is increased attention to the process through which the *observer-independent*, but *family-dependent probabilities* attached to histories may be related to *actual* observer-dependent descriptions of the state of the world.

Finally, let me come to the third strategy (Perspectival Values), whose prime example is the many-worlds interpretation (Everett, 1957; Wheeler, 1957; DeWitt, 1970) and its variants. If the "branching" of the wave function in the many-worlds interpretation is considered as a physical process, it raises, I believe, the very same sort of difficulties as the von Neumann "collapse" does. When does it happen? Which systems are measuring systems that make the world branch? These difficulties of the many-world interpretation have been discussed in the literature (Earman, 1986). Alternatively, we may forget branching as a physical process and keep evolving the wave function under unitary evolution. The problem is then to interpret the observation of the "internal" observers. As discussed in Butterfield (1985) and Albert (1992), this can be done by giving preferred status to special observers (apparatuses) whose values determine a (perspectival) branching. See Objection 7 in Section 2.2 for more details. A natural variant is taking brains—"minds"—as the preferred systems that determine this perspectival branching, and thus whose state determines the new "dimension" of indexicality. Preferred apparatuses, or bringing minds into the game, violates Hypothesis 1.

However, there is a way of having (perspectival) branching keeping all systems on the same footing: the way followed in this paper, namely to assume that all value assignments are completely relational, not just relational with respect to apparatuses or minds. Notice, however, that from this perspective Everett's wave function is a very misleading notion, because not only does it represent the perspective of a nonexistent observer, but it even fails to contain any relevant information about the values observed by any single observer! There is no description of the universe "*in toto*," only a quantum-interrelated net of partial descriptions.

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histories have negligible probability, except the history B (incompatible with A), which has probability close to 1 (perhaps in B all dinosaurs committed suicide). You would like to say, I presume, that B is exactly what has happened. So what has happened? (How did the dinosaurs become extinct?) These curious circumstances, allowed by the history framework, do not invalidate the consistency of the formulation, but put me in the condition of wishing to understand what quantum mechanics is exactly saying about the world, such that these funny circumstances may happen. This paper is a tentative answer.

With respect to Butterfield's classification, the interpretation proposed here is thus in the second, as well as in the third, groups: I maintain that the extra values assigned are "somehow perspectival" (but definitely not mental!), in that they are observer dependent, but at the same time "wholly a matter of physics," in the sense in which the "perspectival" aspect of simultaneity is "wholly a matter of physics" in relativity. In brief: value assignment in a measurement is not inconsistent with unitary evolution of the apparatus + system ensemble, because value assignment refers to the properties of the system *with respect to the apparatus*, while the unitary evolution refers to properties *with respect to an external system.*

From the point of view discussed here, Bohr's interpretation, consistent and coherent histories interpretations, as well as the many-worlds interpretation are all quite literally correct, albeit incomplete. The point of view closest to the one presented here is perhaps Heisenberg's. Heisenberg's insistence on the fact that the lesson to be taken from the atomic experiments is that we should stop thinking of the "state of the system" has been obscured by the subsequent terse definition of the theory in terms of states given by Dirac. Here, I have taken Heisenberg's lesson to some extreme consequences.[7]

Louis Crane is developing a point of view extremely familiar to the one discussed here and has attempted an ambitious extension of these ideas to the cosmological general-covariant gravitational case (Crane, 1995). Finally, it was recently brought to my attention that Zurek ends his paper (Zureck, 1982) with conclusions that are identical to the ones developed here: "Properties of quantum systems have no absolute meaning. Rather, they must be always characterized with respect to other physical systems" and "correlations

[7]With a large number of exceptions, most physicists hold some version of naive realism or some version of naive empiricism. I am aware of the "philosophical qualm" that the ideas presented here may then generate. The conventional reply, which I reiterate here, is that Galileo's relational notion of velocity used to produce analogous qualms, and that physics seems to have the remarkable capacity of challenging even its own conceptual premises in the course of its evolution. Historically, the discovery of quantum mechanics has had a strong impact on the philosophical credo of many physicists, as well as on part of contemporary philosophy. It is possible that this process is not concluded. But I certainly do not want to venture onto philosophical terrain here, and I leave this aspect of the discussion to more competent thinkers. On the other side, the following few observations may perhaps be relevant. The relational aspect of knowledge is of course one of the themes around which part of Western philosophy developed (in Kantian terms, only to mention a characteristic voice, any phenomenal substance which may be an object of possible experience is "entirely made up of mere relations" (Kant, 1787). In recent years, the idea that the notion of "observer-independent description of a system" is meaningless has become almost a commonplace in disparate areas of contemporary culture, from anthropology to certain areas of biology and neurophysiology, from the post-neopositivist tradition to (much more radically) continental philosophy (Gadamer, 1989), all the way to theoretical physical education (Bragagnolo *et al.*, 1993). I find the fact that quantum mechanics, which has directly contributed to inspiring many of these views, has then remained unconnected to these conceptual developments quite curious.

between the properties of quantum systems are more basic that the properties themselves."

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